3B2v8.06a/w (Dec 5 2003).51c XML:ver:5.0.1 CAG : 1478 Prod.Type:COM pp.1–4(col.fig.:1–10) ED:Asha PAGN:Kaushik SCAN:padma

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Chaos and Graphics

Sphere inversion fractals

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Abstract

Three-dimensional fractals consisting of constellations of spheres can be constructed by iterative sphere inversions. Three methods are explored in this brief artistic statement.

Keywords: ■; ■; ■

Multiple methods exist to calculate and represent 3-D fractals. For example, spectacular, artistic images can be obtained using quaternion algebra [1]. In this article, I will briefly discuss three relatively simple methods to calculate fractal shapes consisting only of spheres. All these methods use iterative sphere inversions.

A sphere inversion is the 3-D equivalent of a circle inversion. It is a transformation that maps the outside of a circle to the inside and vice versa. Circle inversions map circles to circles, and fractal shapes can be obtained by iterative inversions of a set of well-chosen initial circles in a set of inversion circles [2]. In Fig. 1, the blue circles are the inversion circles, and the green circles are the initial circles. Figs. 2 and 3 show the result after 1 and 5 iterations, respectively. Note that the initial circles do not overlap and, hence, that none of the calculated circles will overlap. A self-similar fractal pattern of Appolonian circles is generated. The same principle can be applied in three dimensions. A sphere inversion will map a sphere to a sphere. If the initial spheres do not overlap, then neither will any of the calculated spheres. A sphere that is orthogonal to an inversion sphere will not be affected by the inversion transformation.

In the example represented in Figs. 1–3, the inversion circles are on the vertices of a polygon, so the obvious choice for a 3-D constellation is to arrange a set of inversion spheres and initial spheres along the geometry of the platonic solids: the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron [3]. These regular polyhedra have edges of equal length, so that initial spheres can be placed with their center at the vertices, and with a radius equal to half the length of the edge of the polyhedron, so that the initial spheres will be touching. Inversion spheres are placed so that they are orthogonal to the spheres on one face of the polyhedron. One other inversion sphere is placed at the center of the polyhedron, orthogonal to all initial spheres. With this constellation, the iterative inversions will fill the sphere around the initial spheres with ever smaller spheres, and an ideal spherical packing is obtained for all the regular polyhedra, except for the icosahedron [4]. Figs. 4 and 5 show the result for the octahedron, and Fig. 6 is from the dodecahedron. More examples are available at the author's website [5]. The arrangement can be changed so that the inversion spheres, instead of the initial spheres, are placed on the vertices of the polyhedron. Fig. 7 is an example of this arrangement.

Another method may be considered 'pseudo-3-D' as it uses circle inversions in two dimensions, but represents the resulting circles as spheres. Well-chosen configura-

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0097-8493/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.cag.2005.03.011

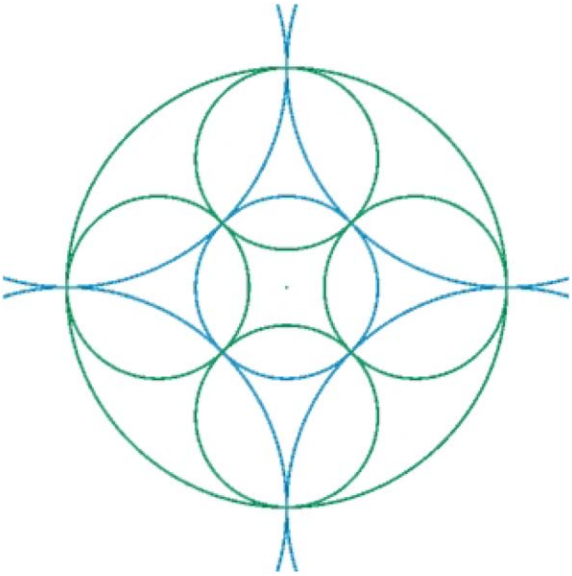

Fig. 1. Circle inversion base configuration.

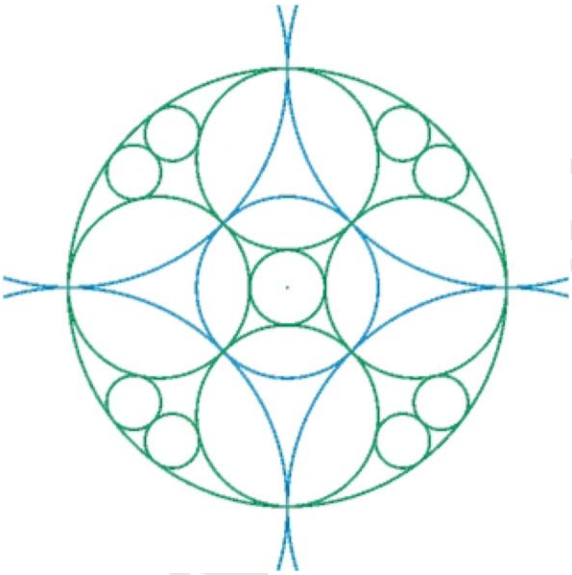

Fig. 2. Circle inversion, one iteration.

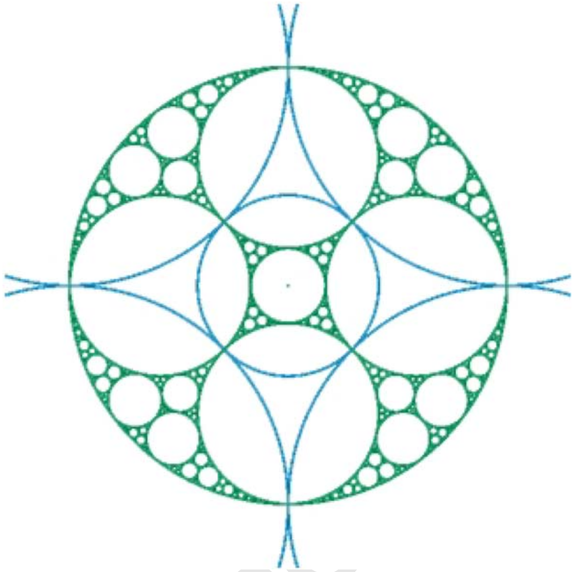

Fig. 3. Circle inversion, five iterations.

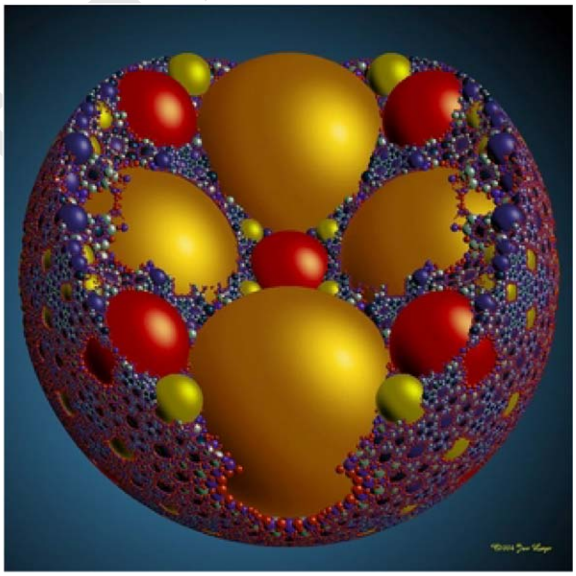

Fig. 4. Octahedron-based packing 1.

tions of initial circles and inversion circles lead to interesting shapes: Fig. 8 is a torus with an infinite number of holes.

Yet another method [6], also 'pseudo-3-D,' was used to compute the images in Figs. 9 and 10. In this method, a combination of inversion, reflection and translation transformations are used to produce a strip of touching circles ('strip geometry') as in Fig. 9. If one additional inversion in a well-chosen sphere is performed that maps this whole strip into one sphere, a configuration as displayed in Fig. 10 is obtained.

All images presented here were made using algorithms written for a software application called Ultrafractal [7]. This program is raster based, which presented some interesting challenges in rendering a large number of spheres (nearly two million in some images) in an

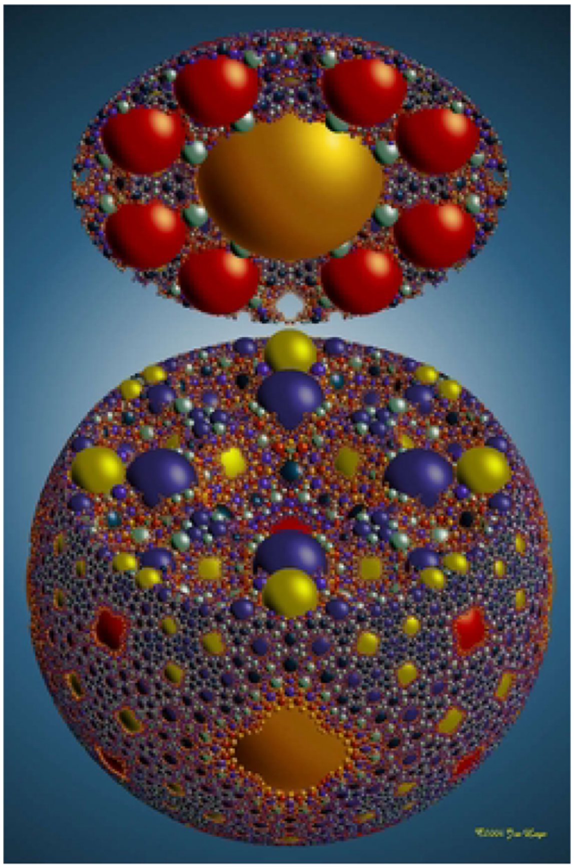

Fig. 5. Octahedron-based packing 2.

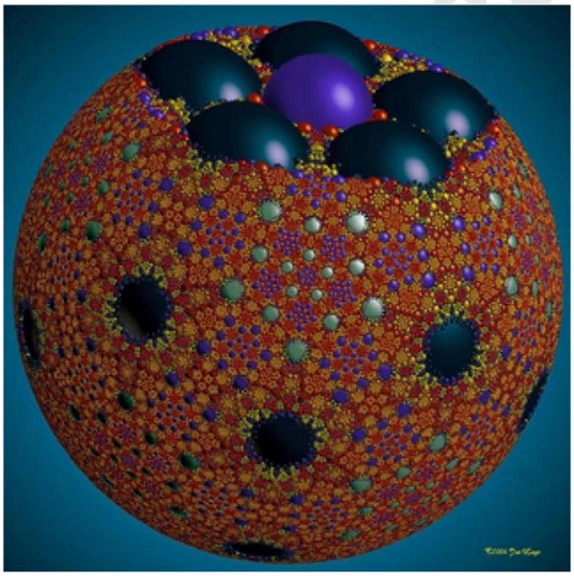

Fig. 6. Dodecahedron-based packing.

Fig. 7. Inversion spheres at vertices of polyhedron.

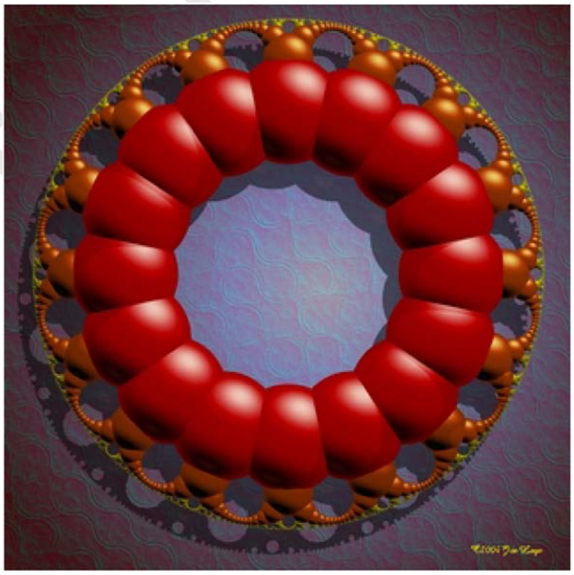

Fig. 8. Infinitely punctured torus.

acceptable timeframe. All of the locations, radii, and base colors of the spheres need to be computed firs. As Ultrafractal allows the use of complex numbers, one can define the spatial position with two complex numbers (*xy* plane and *xz* plane), which simplifies some calculations. Due to the arrangement of inversion spheres, it is possible that some spheres are computed multiple times, and these need to be eliminated, which adds to the

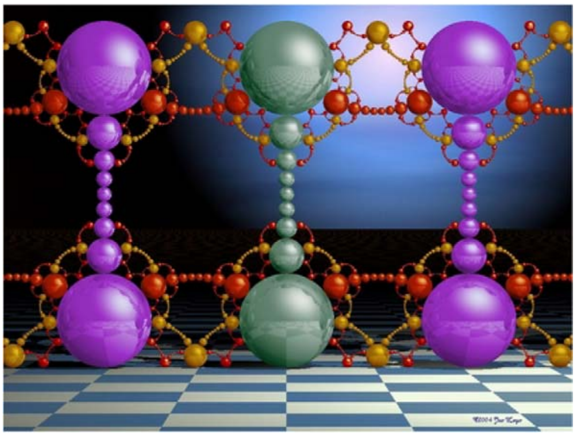

Fig. 9. Strip geometry.

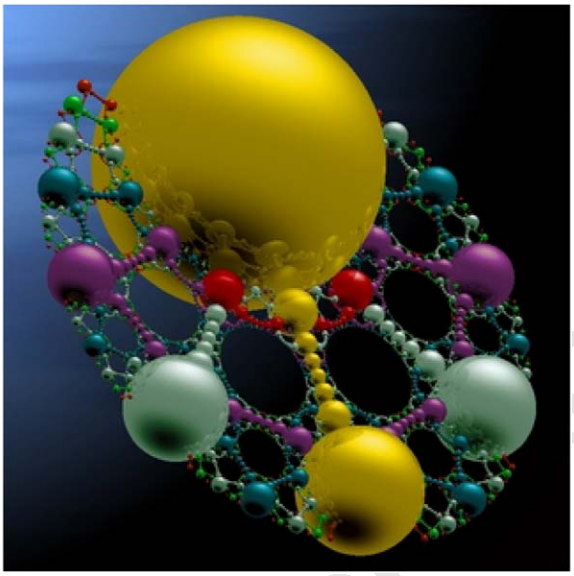

Fig. 10. Strip geometry transformed to one sphere.

computation time. This challenge was overcome by classifying the computed spheres in an array representing a 3-D grid, so that the time needed for comparison with already computed spheres is drastically reduced. For final onscreen rendering, all spheres are classified in another 2-D grid, in order to reduce the calculation time of determining the frontmost sphere. The raytracing routine used in some of the images was written as part of the algorithm because the program does not have a separate module for this.

In this brief artistic note, I have explored some of the possibilities to obtain fractal patterns with just spheres. Fairly simple algorithms can produce very interesting and appealing shapes.

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